8. G. I. Barenblatt, Similarity, Self-Similarity and Intermediate Asymptotics [in Russian], Leningrad (1982).
9. V. P. Kochergin, V. I. Klimov, and V. A. Sukhorukov, Chislennye Metody Mekh. Sploshnoi Sredy, 6, No. 5, 72-84 (1976).
10. M. I. Zheleznyak, in: Hydromechanics [in Russian], No. 41 (1980), pp. 72-78.
11. V. S. Maderich, in: Hydromechanics [in Russian], No. 46 (1982), pp. 80-84.

FLOW AND HEAT TRANSFER OF FINELY DISPERSED TURBULENT FLOWS IN CHANNELS

I. V. Derevich, V. M. Eroshenko, and L. I. Zaichik

UDC 532.529

Equations for the second moments of the velocity and temperature fluctuations are used to study the effect of particles on the rate of turbulent momentum and heat transfer in the flow of a gas suspension in circular pipes.

It is currently most promising to describe the hydrodynamics and heat transfer of turbulent disperse flows by using the system of equations for the second one-point moments of the velocity and temperature fluctuations of the dispersion medium with allowance for the presence of the particles [1-4]. The authors of [5-8] used this system to analyze the effect of the disperse phase on the fluctuation and mean flow and heat-transfer characteristics of dust-laden flows in channels for particles for which the dynamic and thermal relaxation times were of the same order as the integral turbulence scale. The present investigation, being a continuation of [5-8], studies the manner in which the rate of turbulent transport is affected by finer particles, having a dynamic relaxation time which is one order less than the microscopic time scale of the turbulence. We will also present results of calculations of the hydrodynamics and heat transfer of dust-laden flows within a broad range of particle dimensions.

1. We are examining the turbulent flow of a gas with spherical solid particles ($\rho_2 \gg \rho_1$). The system of equations of motion and heat transfer of the gas in the case of a small volume content of particles has the form:

$$\frac{\partial U_i}{\partial t} + U_k \frac{\partial U_i}{\partial x_k} = -\frac{1}{\rho_1} \frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_k \partial x_k} - \frac{\rho_2}{\rho_1} \frac{\omega}{\Omega_N} \sum_{p=1}^{N} \delta(\mathbf{x} - \mathbf{R}_p(t)) \frac{dV_{pi}(t)}{dt}, \quad (1)$$

$$\frac{\partial \Theta_1}{\partial t} + U_k \frac{\partial \Theta_1}{\partial x_k} = \chi \frac{\partial^2 \Theta_1}{\partial x_k \partial x_k} - \frac{\rho_2 c_2}{\rho_1 c_1} \frac{\omega}{\Omega_N} \sum_{p=1}^{N} \delta(\mathbf{x} - \mathbf{R}_p(t)) \frac{d\Theta_p(t)}{dt}, \quad (2)$$

$$\frac{dV_{pi}}{dt} = \frac{1}{\tau_u}(U_i(\mathbf{R}_p(t), t) - V_{pi}(t)), \quad \frac{dR_{pi}}{dt} = V_{pi}, \quad (3)$$

$$\frac{d\Theta_p}{dt} = \frac{1}{\tau_\theta}(\Theta_1(\mathbf{R}_p(t), t) - \Theta_p(t)). \quad (4)$$

If we change over from a Lagrangian description of the individual particles (3), (4) to an Eulerian description of the solid phase [8], average Eqs. (1) and (2) and the equations obtrained for the solid phase in the case of turbulent flow, and add up the equations for the individual phases, we obtain the equations of motion and heat transfer for the disperse flow as a whole:

G. M. Krzhizhanovskii Scientific-Research Institute of Power Engineering, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 53, No. 5, pp. 740-751, November, 1987. Original article submitted March 5, 1987.

 0022-0841/87/5305-1252 $12.50

$$\frac{\partial \langle U_i \rangle}{\partial t} + \langle U_k \rangle \frac{\partial \langle U_i \rangle}{\partial x_k} + \langle C \rangle \frac{\rho_2}{\rho_1}\left(\frac{\partial \langle V_i \rangle}{\partial t} + \langle V_k \rangle \frac{\partial \langle V_i \rangle}{\partial x_k}\right) = \tag{5}$$

$$= -\frac{1}{\rho_1}\frac{\partial \langle P \rangle}{\partial x_i} + \frac{\partial}{\partial x_k}\left(\nu \frac{\partial \langle U_i \rangle}{\partial x_k} - \langle u_i u_k \rangle - \langle C \rangle \frac{\rho_2}{\rho_1} \langle v_i v_k \rangle\right),$$

$$\frac{\partial \langle \Theta_1 \rangle}{\partial t} + \langle U_k \rangle \frac{\partial \langle \Theta_1 \rangle}{\partial x_k} + \langle C \rangle \frac{c_2\rho_2}{c_1\rho_1}\left(\frac{\partial \langle \Theta_2 \rangle}{\partial t} + \langle V_k \rangle \frac{\partial \langle \Theta_2 \rangle}{\partial x_k}\right) =$$

$$= \frac{\partial}{\partial x_k}\left[\chi \frac{\partial \langle \Theta_1 \rangle}{\partial x_k} - \langle \theta_1 u_k \rangle - \frac{c_2\rho_2}{c_1\rho_1} \langle C \rangle \langle \theta_2 v_k \rangle\right]. \tag{6}$$

It is evident from Eqs. (5) and (6) that particles brought into pulsative motion by the carrier phase increase the rate of turbulent transfer of momentum and heat.

2. To close the averaged equations of motion and heat transfer (5) and (6), it is necessary to obtain expressions for the second one-point moments of the velocity and temperature fluctuations of the carrier and solid phases. The equations for the carrier phase, with allowance for the presence of the particles, coincide with the corresponding equations for a pure gas [9] except for the additive terms which account for phase interaction. The terms describing the effect of the particles in the equations for the second moments of the velocity fluctuations, the correlations of velocity and temperature fluctuations, and the square of the temperature fluctuations of the carrier phase, respectively, have the form:

$$\varepsilon_{ij}^p = \frac{\rho_2}{\rho_1}\frac{1}{\tau_u}\left\{2\langle C \rangle \langle u_i u_j \rangle - \left\langle \frac{\omega}{\Omega_N}\sum_{p=1}^{N} \delta(\mathbf{x} - \mathbf{R}_p(t))\,[V_{pi}(t)\,u_j(\mathbf{x}, t) + V_{pj}(t)\,u_i(\mathbf{x}, t)]\right\rangle\right\}, \tag{7}$$

$$\varepsilon_{\theta i}^p = \frac{\rho_2}{\rho_1}\left\{\langle C \rangle\left[\frac{c_2}{c_1}\frac{\langle \theta_1 u_i \rangle}{\tau_\theta} + \frac{\langle \theta_1 u_i \rangle}{\tau_u}\right] - \left\langle \frac{\omega}{\Omega_N}\sum_{p=1}^{N} \delta(\mathbf{x} - \mathbf{R}_p(t))\left[\frac{c_2}{c_1}\frac{\Theta_p(t)\,u_i(x, t)}{\tau_\theta} + \frac{\theta_1(x, t)\,V_{pi}(t)}{\tau_u}\right]\right\rangle\right\}, \tag{8}$$

$$\varepsilon_\theta^p = \frac{2}{\tau_\theta}\frac{\rho_2 c_2}{\rho_1 c_1}\left\{\langle C \rangle \langle \theta_1^2 \rangle - \left\langle \frac{\omega}{\Omega_N}\sum_{p=1}^{N} \delta(\mathbf{x} - \mathbf{R}_p(t))\,\Theta_p(t)\,\theta_1(\mathbf{x}, t)\right\rangle\right\}. \tag{9}$$

The velocity and temperature of the particles are found from Eqs. (3) and (4) and, at $t \gg \tau_u$, τ_θ they have the form:

$$V_{pi}(t) = \frac{1}{\tau_u}\int_0^t ds \exp\left(-\frac{t-s}{\tau_u}\right) U_i(\mathbf{R}_p(s), s), \tag{10}$$

$$\Theta_p(t) = \frac{1}{\tau_\theta}\int_0^t ds \exp\left(-\frac{t-s}{\tau_\theta}\right) \Theta_1(\mathbf{R}_p(s), s). \tag{11}$$

Isolating the average and fluctuation components of velocity and temperature for the carrier phase in Eqs. (10) and (11) and using the identity

$$\int_0^t \varphi(x(s), s)\,ds = \int_0^t ds\varphi(x(t), s) - \int_0^t ds \int_0^s ds_1 \frac{dx(s)}{ds}\frac{\partial \varphi(x(s), s_1)}{\partial x}$$

and in addition considering that the scales of the change in the mean quantities are considerably greater than the scales of change in the fluctuation velocities, we obtain the following expressions for the terms entering into (7)-(9):

$$\langle \delta(\mathbf{x} - \mathbf{R}_p(t))\,V_{pi}(t)\,u_j(\mathbf{x}, t) \rangle = \langle \delta(\mathbf{x} - \mathbf{R}_p(t))\,u_j(x, t) \rangle \times$$

$$\times \left(\langle U_i \rangle - \tau_u \langle V_k \rangle \frac{\partial \langle U_i \rangle}{\partial x_k}\right) - \frac{1}{\tau_u}\int d\mathbf{x}_1 \int_0^t ds\,(t-s) \times$$

$$\times \exp\left(-\frac{t-s}{\tau_u}\right) \langle \delta(\mathbf{x} - \mathbf{R}_p(t))\,\delta(\mathbf{x}_1 - \mathbf{R}_p(s))\,u_k(\mathbf{x}_1, s)\,u_j(\mathbf{x}, t) \rangle \times \tag{12}$$

$$\times \frac{\partial \langle U_i \rangle}{\partial x_k} + \frac{1}{\tau_u}\int d\mathbf{x}_1 \int_0^t ds \exp\left(-\frac{t-s}{\tau_u}\right) \langle \delta(\mathbf{x} - \mathbf{R}_p(t)) \times$$

$$\times \delta(\mathbf{x}_1 - \mathbf{R}_p(s))\,u_i(\mathbf{x}_1, s)\,u_j(\mathbf{x}, t) \rangle,$$

$$
\begin{aligned}
&\langle \delta(\mathbf{x}-\mathbf{R}_p(t))\,\Theta_p(t)\,u_i(\mathbf{x},\ t)\rangle = \langle \delta(\mathbf{x}-\mathbf{R}_p(t))\,u_i(\mathbf{x},\ t)\rangle \times \\
&\times\left(\langle \Theta_1\rangle - \tau_\theta \langle V_k\rangle \frac{\partial\langle \Theta_1\rangle}{\partial x_k}\right) - \frac{1}{\tau_u-\tau_\theta}\int d\mathbf{x}_1 \int_0^t ds\,\times \\
&\times\left[\exp\left(-\frac{t-s}{\tau_u}\right) - \exp\left(-\frac{t-s}{\tau_\theta}\right)\right]\langle \delta(\mathbf{x}-\mathbf{R}_p(t))\,\delta(\mathbf{x}_1 - \\
&-\mathbf{R}_p(s))\,u_k(\mathbf{x}_1,\ s)\,u_i(\mathbf{x},\ t)\rangle \frac{\partial\langle \Theta_1\rangle}{\partial x_k} + \frac{1}{\tau_\theta}\int d\mathbf{x}_1 \int_0^t ds\,\times \\
&\times \exp\left(-\frac{t-s}{\tau_\theta}\right)\langle \delta(\mathbf{x}-\mathbf{R}_p(t))\,\delta(\mathbf{x}_1-\mathbf{R}_p(s))\,\times \\
&\times\,\theta_1(\mathbf{x}_1,\ s)\,u_i(\mathbf{x},\ t)\rangle,
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
&\langle \delta(\mathbf{x}-\mathbf{R}_p(t))\,V_{pi}(t)\,\theta_1(\mathbf{x},\ t)\rangle = \langle \delta(\mathbf{x}-\mathbf{R}_p(t))\,\theta_1(\mathbf{x},\ t)\rangle \times \\
&\times\left(\langle U_i\rangle - \tau_u\langle V_k\rangle \frac{\partial\langle U_i\rangle}{\partial x_k}\right) - \frac{1}{\tau_u}\int d\mathbf{x}_1 \int_0^t ds\,(t-s)\,\times \\
&\times \exp\left(-\frac{t-s}{\tau_u}\right)\langle \delta(\mathbf{x}-\mathbf{R}_p(t))\,\delta(\mathbf{x}_1-\mathbf{R}_p(s))\,\times \\
&\times u_i(\mathbf{x}_1,\ s)\,\theta_1(\mathbf{x},\ t)\rangle \frac{\partial\langle U_i\rangle}{\partial x_k} + \frac{1}{\tau_u}\int d\mathbf{x}_1 \int_0^t ds\,\times \\
&\times \exp\left(-\frac{t-s}{\tau_u}\right)\langle \delta(\mathbf{x}-\mathbf{R}_p(t))\,\delta(\mathbf{x}_1-\mathbf{R}_p(s))\,\times \\
&\times u_i(\mathbf{x}_1,\ s)\,\theta_1(\mathbf{x},\ t)\rangle,
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
&\langle \delta(\mathbf{x}-\mathbf{R}_p(t))\,\Theta_p(t)\,\theta_1(\mathbf{x},\ t)\rangle = \langle \delta(\mathbf{x}-\mathbf{R}_p(t))\,\times \\
&\times \theta_1(\mathbf{x},\ t)\rangle\left(\langle \Theta_1\rangle - \tau_\theta\langle V_k\rangle \frac{\partial\langle \Theta_1\rangle}{\partial x_k}\right) - \\
&-\frac{1}{\tau_u-\tau_\theta}\int d\mathbf{x}_1 \int_0^t ds\left[\exp\left(-\frac{t-s}{\tau_u}\right) - \exp\left(-\frac{t-s}{\tau_\theta}\right)\right]\times \\
&\times\langle \delta(\mathbf{x}-\mathbf{R}_p(t))\,\delta(\mathbf{x}_1-\mathbf{R}_p(s))\,\theta_1(\mathbf{x}_1,\ s)\,\theta_1(\mathbf{x},\ t)\rangle \\
&\times\frac{\partial\langle \Theta_1\rangle}{\partial x_k} + \frac{1}{\tau_\theta}\int d\mathbf{x}_1 \int_0^t ds\,\exp\left(-\frac{t-s}{\tau_\theta}\right)\langle \delta(\mathbf{x}-\mathbf{R}_p(t))\,\times \\
&\times\,\delta(\mathbf{x}_1-\mathbf{R}_p(s))\,\theta_1(\mathbf{x}_1,\ s)\,\theta_1(\mathbf{x},\ t)\rangle.
\end{aligned}
\tag{15}
$$

It is evident from Eqs. (12)-(15) that the rate of phase interaction is determined by the Lagrangian correlations of the velocity and temperature fluctuations of the gas phase calculated from the trajectory of the particles. The first terms in the right sides of (12)-(15) are proportional to the gradient of the mean concentration of the discrete phase. To determine the remaining terms, we can examine the Lagrangian correlation of the velocity fluctuations of the gas phase according to the particle trajectory:

$$
\langle F_{ij}(\mathbf{x},\ \mathbf{x}_1;\ t,\ s)\rangle = \langle \delta(\mathbf{x}-\mathbf{R}_p(t))\,\delta(\mathbf{x}_1-\mathbf{R}_p(s))\,[u_i(\mathbf{x},\ s)\,u_j(\mathbf{x},\ t) + u_j(\mathbf{x}_1,\ s)\,u_i(\mathbf{x},\ t)]. \tag{16}
$$

In a nonuniform turbulent flow, the function $\langle F_{ij} \rangle$ depends not only on the relative variables $z = x - x_1$ and $\xi = t - s \geq 0$ – the scales of which coincide with the scales of change of the fluctuation variables – but it also depends on the variables $x_0 = (x + x_1)/2$ and $t_0 = (t + s)/2$, characterizing the average change of $\langle F_{ij} \rangle$ in the flow.

Considering that the scales of the variables x_0 and t_0 appreciably exceed the scales of the variables z and ξ, we put:

$$\langle F_{ij}(\mathbf{x}, \mathbf{x}_1; t, s) \rangle = \langle F_{ij}(\mathbf{x}_0, t_0; \mathbf{z}, s) \rangle = \\ = \left\langle F_{ij}\left(\mathbf{x} - \frac{\mathbf{z}}{2}, t - \frac{\xi}{2}; \mathbf{z}, s\right)\right\rangle \approx \langle F_{ij}(\mathbf{x}, t; \mathbf{z}, \xi) \rangle - \frac{\xi}{2} \times \\ \times \left[\frac{\partial \langle F_{ij}(\mathbf{x}, t; \mathbf{z}, \xi) \rangle}{\partial t} + \left\langle \frac{\partial F_{ij}(\mathbf{x}, t; \mathbf{z}, \xi)}{\partial x_k} \frac{\partial z_k}{\partial \xi} \right\rangle\right]. \tag{17}$$

The first term in the square brackets describes the mean change over time in the rate of turbulent pulsations of the carrier phase, while the second term describes convective transfer and turbulent diffusion of the pulsations in the nonuniform flow. For the Lagrangian correlation $\langle F_{ij}(x, t; z, \xi) \rangle$, we determine the macroscopic integral time scale T_{Ep} from the following relation:

$$\int d\mathbf{z} \int_0 d\xi \langle F_{ij}(\mathbf{x}, t; \mathbf{z}, \xi) \rangle = 2T_{Ep} \langle n_1(\mathbf{x}, t) \rangle \langle u_i u_j \rangle .$$

The time macroscale T_{Ep} differs from the integral time scale of turbulence T_E – equal to the lifetime of energy-containing moles – due to the averaged and fluctuational slip of the phases. Ignoring the averaged phase slip and considering that the fluctuational phase slip does not lead to a substantial reduction in T_{Ep} [1], for simplicity we will henceforth assume that $T_{Ep} = T_E$.

Most of the turbulent energy of the flow lies in the energy-containing moles, the characteristic lifetime of which is on the order of the time macroscale of turbulence T_E. At the same time, the dissipation of the pulsative energy is connected with high-frequency pulsations, the period of these pulsations being on the order of the Taylor microscale T_0. Meanwhile, $T_0 \ll T_E$ for large Reynolds numbers [9]. Since particles having a dynamic relaxation time greater than or of the same order as the integral turbulence scale do not react to small-scale pulsations having a lifetime much less than T_E and since only energy-containing moles are brought into pulsative motion, the Lagrangian correlation for them $\langle F_{ij}(x, t, z, \xi) \rangle$ can be approximated by the step function

$$\langle F_{ij}(\mathbf{x}, t; \mathbf{z}, \xi) \rangle = \begin{cases} 2 \langle n_1(\mathbf{x}, t) \rangle \langle u_i u_j \rangle \delta(\mathbf{z}) & \text{at} \quad \xi \leqslant T_E, \\ 0 & \text{at} \quad \xi > T_E. \end{cases}$$

As a result, we obtain the following for the integral terms in (12):

$$\frac{1}{\tau_u} \int d\mathbf{z} \int_0^\infty d\xi \exp\left(\frac{\xi}{\tau_u}\right) \langle F_{ij}(\mathbf{x}, t, \mathbf{z}, \xi) \rangle = 2f_{u1} \langle n_1 \rangle \langle u_i u_j \rangle , \\ \frac{1}{\tau_u^2} \int d\mathbf{z} \int_0^\infty d\xi \xi \exp\left(-\frac{\xi}{\tau_u}\right) \langle F_{ij}(\mathbf{x}, t; \mathbf{z}, \xi) \rangle = 2f_{u2} \langle n_1 \rangle \langle u_i u_j \rangle , \tag{18}$$

where $f_{u1} = 1 - \exp(-T_E/\tau_u)$, $f_{u2} = 1 - (1 + T_E/\tau_u)\exp(-T_E/\tau_u)$. The functions f_{u1} and f_{u2} describe the degree of entrainment of the particles in pulsative motion and coincide with the functions obtained earlier in [5].

The fine particles ($\tau_u \lessapprox T_0$) are completely entrained in the turbulent motion of the energy-containing moles of the carrier phase; here, effects connected with fluctuational slip of these moles and the solid phase can be ignored. Effects due to the participation of particles in the high-frequency small-scale motion of the carrier phase become important for particles with a relaxation time which is less than the Taylor microscale of turbulence. This fact leads in particular to an increase in turbulent dissipation in a dust-laden flow [10]. In the case $\tau_u < T_0$ the upper limit in the integral terms in Eq. (12) is assumed to be equal to T_0. Then taking into consideration the equality $\partial \langle F_{ij}(x, t; z, \xi) \rangle / \partial \xi = 0$ at $\xi = 0$ [11] and considering that the range of the variable z is associated with the inertial interval, we will change over to a spectral representation for the equation of motion of the carrier phase (1) and the expression of the Lagrangian correlation of fluctuation velocity (17). As a result, we find (for example) the following expression for the last term in (12) at $\tau_u < T_0$

$$\frac{1}{\tau_u}\int dz \int_0^{T_0} d\xi \exp\left(-\frac{\xi}{\tau_u}\right)\langle F_{ij}(\mathbf{x},\ t;\ \mathbf{z},\ \xi)\rangle =$$

$$= \langle n_1\rangle\left[1-\exp\left(-\frac{T_0}{\tau_u}\right)\right]\int d\mathbf{k}\left\{\frac{2E_{ij}(\mathbf{k})+\tau_u\pi_{ij}(\mathbf{k})}{1+\tau_u\nu k^2}-\right.$$
$$\left.-\frac{\tau_u}{(1+\tau_u\nu k^2)^2}\left[E_{in}(\mathbf{k})\frac{\partial\langle U\rangle}{\partial x_n}+E_{jn}(\mathbf{k})\frac{\partial\langle U_i\rangle}{\partial x_k}\right]\right\}, \tag{19}$$

where $E_{ij}(\mathbf{k})$ and $\pi_{ij}(\mathbf{k})$ are the Fourier components of velocity and pressure fluctuations in the carrier phase, determined from the relations

$$\langle u_i(\mathbf{x}+\mathbf{y},\ t)\,u_j(\mathbf{x},\ t)\rangle = \int d\mathbf{k}\exp(i\mathbf{ky})E_{ij}(\mathbf{k}),$$

$$-\frac{1}{\rho_1}\left\langle\frac{\partial p(\mathbf{x}+\mathbf{y},\ t)\,u_i(\mathbf{x},\ t)}{\partial x_j}+\frac{\partial p(\mathbf{x}+\mathbf{y},\ t)\,u_j(\mathbf{x},\ t)}{\partial x_i}\right\rangle = \int d\mathbf{k}\exp(i\mathbf{ky})\,\pi_{ij}(\mathbf{k}).$$

The integral term in (12) with gradients of averaged velocities is proportional to terms on the order of $(\tau_u/T_0)^2$ and is henceforth omitted.

Thus, we have two representations for the integral terms in Eq. (12): expressions (18), valid at $\tau_u \sim T_E$, and expression (19), for particles with $\tau_u < T_0 \ll T_E$. Considering that most of the energy of the pulsative motion is concentrated in large-scale eddies, comparison of Eqs. (18) and (19) in the region $T_0 \lessgtr \tau_u \lessgtr T_E$ yields a closed expression for correlations of the particle velocities with fluctuations of gas velocity in (12)

$$J_{ij} = \langle\delta(\mathbf{x}-\mathbf{R}_p(t))[V_{pi}(t)\,u_j(\mathbf{x},\ t)+V_{pj}(t)\,u_i(\mathbf{x},\ t)]\rangle =$$
$$= \langle\delta(\mathbf{x}-\mathbf{R}_p(t))\,u_j(\mathbf{x},\ t)\rangle\left[\langle U_i\rangle-\tau_u\langle V_k\rangle\frac{\partial\langle U_i\rangle}{\partial x_k}\right]+$$
$$+\langle\delta(\mathbf{x}-\mathbf{R}_p(t))\,u_i(\mathbf{x},\ t)\rangle\left[\langle U_j\rangle-\tau_u\langle V_k\rangle\frac{\partial\langle U_j\rangle}{\partial x_k}\right]+$$
$$+\langle n_1\rangle\left[2f_{u1}\langle u_iu_j\rangle-\tau_uf_{u2}\left(\langle u_iu_k\rangle\frac{\partial\langle U_j\rangle}{\partial x_k}+\right.\right.$$
$$\left.+\langle u_ju_k\rangle\frac{\partial\langle U_i\rangle}{\partial x_k}+\frac{\partial\langle u_iu_j\rangle}{\partial t}\right)-\tau_uf_{u3}\frac{\partial\langle u_iu_ju_k\rangle}{\partial x_k}-$$
$$\left.-f_{u1}^0\int\frac{d\mathbf{k}}{1+\tau_u\nu k^2}(2\tau_u\nu k^2E_{ij}(\mathbf{k})+\tau_u\pi_{ij}(\mathbf{k}))\right], \tag{20}$$

where

$$f_{u1}^0 = 1-\exp\left(-\frac{T_0}{\tau_u}\right),\quad f_{u3} = 1-\left[1+\frac{T_E}{\tau_u}+\frac{1}{2}\left(\frac{T_E}{\tau_u}\right)^2\right]\exp\left(-\frac{T_E}{\tau_u}\right).$$

The first two terms in the right side of Eq. (20) are proportional to the gradient of the mean particle concentration. The terms in the brackets, except for the integral term, describe effects connected with entrainment of particles in the large-scale pulsations of the energy-containing moles and coincide with the terms obtained earlier in [5]. The integral term in (20) describes effects connected with participation of the particles in small-scale turbulent motion.

Let us examine a statistically uniform and steady turbulent flow. Summing over i = j in the approximation of constant particle concentration, we obtain the following from Eq. (20)

$$J_{ii} = 2\langle n_1\rangle\left(f_{u1}\langle u_iu_j\rangle-f_{u1}^0\int d\mathbf{k}\frac{\tau_u\nu k^2E_{ii}(\mathbf{k})}{1+\tau_u\nu k^2}\right). \tag{21}$$

It follows from (21) that $J_{ii} \approx 2 \langle n_1 \rangle \langle u_i u_i \rangle T_E/\tau_u$ for particles with a relaxation time considerably greater than the integral scale of turbulence. This result coincides with the results obtained in [1, 2, 5]. For finer particles, $T_0 < \tau_u \ll T_E$, we have

$$J_{ii} \approx 2\left(\langle u_i u_i \rangle - \frac{T_0}{\tau_u} \int d\mathrm{k}\, \frac{\tau_u \nu k^2 E_{ii}(\mathrm{k})}{1+\tau_u \nu k^2} \right).$$

It should be noted that $2\nu \int d\mathrm{k} k^2 E_{ii}(\mathrm{k}) = \varepsilon$, while the maximum of the expression $k^2 E_{ii}(k)$ in the wave-number space lies in the neighborhood $k \sim 1/\ell_\eta$ [9, 11] (ℓ_η is the Kolmogorov spatial scale of turbulence) and $\nu \ell_\eta^{-2} = 1/T_\eta$, where T_η is the Kolmogorov time scale, $T_0 > T_\eta$. Then for particles with a relaxation time less than the microscale T_0, it follows from Eq. (21) that

$$J_{ii} \approx 2 \langle u_i u_i \rangle \left(1 - \frac{\tau_u/T_\eta}{1+\tau_u/T_\eta}\right) = 2 \langle u_i u_i \rangle \frac{1}{1+\tau_u/T_\eta}. \tag{22}$$

It is evident from Eq. (21) that to completely involve the particles in the turbulent motion of the carrier phase, the relaxation time of the particles must be less than the Kolmogorov time scale.

Similarly, correlations of the particle velocity and temperature with fluctuations of velocity and temperature of the carrier phase are expressed through one-point moments of the velocity and temperature fluctuations of the carrier phase. Having inserted the resulting expressions into Eqs. (7)-(9), we write the system of equations for the second moments of the velocity and temperature fluctuations and for the square of the temperature fluctuations of the carrier phase in the steady turbulent flow of a gas with a constant particle concentration:

$$\left[\langle U_k \rangle + \frac{\rho_2}{\rho_1} \langle C \rangle f_{u2} \langle V_k \rangle \right] \frac{\partial \langle u_i u_j \rangle}{\partial x_k} + \left(1 + \frac{\rho_2}{\rho_1} \langle C \rangle f_{u2}\right) \times$$
$$\times \left[\langle u_i u_k \rangle \frac{\partial \langle U_j \rangle}{\partial x_k} + \langle u_j u_k \rangle \frac{\partial \langle U_i \rangle}{\partial x_k} \right] + \left(1 + \frac{\rho_2}{\rho_1} \langle C \rangle f_{u3}\right) \times$$
$$\times \frac{\partial \langle u_i u_j u_k \rangle}{\partial x_k} = \left(1 + \frac{\rho_2}{\rho_1} \langle C \rangle g_{u1}\right) \left[\left\langle \frac{p}{\rho_1} \left(\frac{\partial u_i}{\partial x_j} + \right.\right.\right. \tag{23}$$
$$\left.\left. + \frac{\partial u_j}{\partial x_i} \right) \right\rangle - 2\nu \left\langle \frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k} \right\rangle - \frac{1}{\rho_1} \left(\frac{\partial \langle u_i p \rangle}{\partial x_j} + \right.$$
$$\left.\left. + \frac{\partial \langle u_j p \rangle}{\partial x_i} \right) \right] + \nu \frac{\partial^2 \langle u_i u_j \rangle}{\partial x_k \partial x_k} - 2 \frac{\rho_2}{\rho_1} \langle C \rangle f_{u4} \frac{\langle u_i u_j \rangle}{T_E},$$

$$\left[\langle U_k \rangle + \frac{1}{2} \frac{\rho_2}{\rho_1} \langle C \rangle \left(f_{u2} + \frac{C_2}{C_1} f_{\theta u2} \right) \langle V_k \rangle \right] \frac{\partial \langle \theta_1 u_i \rangle}{\partial x_k} +$$
$$+ \left(1 + \frac{\rho_2}{\rho_1} \langle C \rangle f_{u2}\right) \langle \theta_1 u_k \rangle \frac{\partial \langle U_i \rangle}{\partial x_k} + \left(1 + \frac{\rho_2 c_2}{\rho_1 c_1} \langle C \rangle f_{\theta u2}\right) \times \tag{24}$$
$$\times \langle u_i u_k \rangle \frac{\partial \langle \Theta_1 \rangle}{\partial x} + \left[1 + \frac{1}{2} \frac{\rho_2}{\rho_1} \langle C \rangle \left(f_{u3} + \frac{c_2}{c_1} f_{\theta u3} \right) \right] \times$$
$$\times \frac{\partial \langle \theta_1 u_i u_k \rangle}{\partial x_k} = -\left[1 + \frac{\rho_2}{\rho_1} \langle C \rangle \left(g_{u1} + \frac{c_2}{c_1} g_{\theta 1} \right) \right] \times$$
$$\times \left\langle \theta_1 \frac{\partial p}{\partial x_i} \right\rangle + \chi \left\langle \frac{\partial^2 \theta_1}{\partial x_k \partial x_i} u_i \right\rangle + \nu \left\langle \theta_1 \frac{\partial^2 u_1}{\partial x_k \partial x_k} \right\rangle -$$
$$- \frac{\rho_2}{\rho_1} \langle C \rangle \left(f_{u4} + \frac{c_2}{c_1} f_{\theta 4} \right) \frac{\langle \theta_1 u_i \rangle}{T_E},$$

$$\left[\langle U_k\rangle+\frac{\rho_2c_2}{\rho_1c_1}\langle C\rangle f_{\theta2}\langle V_k\rangle\right]\frac{\partial\langle\theta_1^2\rangle}{\partial x_k}+\left(1+\right.$$
$$\left.+\frac{\rho_2c_2}{\rho_1c_1}f_{\theta u2}\langle C\rangle\right)\langle\theta_1u_k\rangle\frac{\partial\langle\Theta_1\rangle}{\partial x_k}+\left(1+\frac{\rho_2c_2}{\rho_1c_1}\langle C\rangle f_{\theta u3}\right)\times$$
$$\times\frac{\partial\langle\theta_1^2u_k\rangle}{\partial x_k}=\chi\frac{\partial^2\langle\Theta_1^2\rangle}{\partial x_k\partial x_k}-\left(1+\frac{\rho_2c_2}{\rho_1c_1}g_{\theta1}\right)\times \tag{25}$$
$$\times\chi\left\langle\left(\frac{\partial\theta_1}{\partial x_k}\right)^2\right\rangle-2\frac{\rho_2c_2}{\rho_1c_1}\langle C\rangle f_{\theta4}\frac{\langle\theta_1^2\rangle}{T_E},$$

where

$$g_{u1}=[1-\exp(-T_0/\tau_u)]/(1+\tau_u/T_\eta),\ g_{\theta1}=[1-\exp(-T_0/\tau_\theta)]/(1+\mathrm{Pr}\tau_\theta/T_\eta),$$
$$f_{\theta u2}=1-[\exp(-1/\Omega_u)\Omega_u-\exp(-1/\Omega_\theta)\Omega_\theta](\Omega_u-\Omega_\theta)^{-1},\ f_{\theta2}=1-(1+1/\Omega_\theta)\times$$
$$\times\exp(-1/\Omega_\theta),\ f_{u4}=\exp(-1/\Omega_u)/\Omega_u,\ f_{\theta4}=\exp(-1/\Omega_\theta)/\Omega_\theta.$$

It can be seen from Eqs. (23)-(25) that the participation of the particles in pulsative motion leads to a change in the terms describing convective transfer, turbulent diffusion, and the generation of turbulent pulsations, as well as the terms describing turbulent dissipation and exchange interaction. Also, new terms appear. These terms account for the additional dissipation of pulsations of the gas due to fluctuational slip of energy-containing moles of the solid and carrier phases; similar dissipative terms (with somewhat different expressions for f_{u4} and $f_{\theta4}$) were obtained earlier in [1, 2]. The character of the effect of the discrete phase on the rate of pulsative motion of the carrier phase is determined by the degree of involvement of the particles in the pulsative motion. Highly inertial particles (τ_u, $\tau_\theta \gg T_E$) do not have a significant effect on the fluctuation characteristics of the carrier flow, and for them the functions f and g approach zero. Particles having a relaxation time on the order of the time macroscale of turbulence (τ_u, $\tau_\theta \sim T_E$, g_{u1}, $g_{\theta1} \ll 1$, the functions f $\sim$ 1) participate mainly in the turbulent motion of the energy-containing moles; here, on the one hand, the participation of particles in turbulent motion leads to an increase in the generation of fluctuation energy of the flow, while on the other hand the turbulent energy of the carrier phase is expended on entraining the disperse phase in pulsative motion. The difference between the additional generation of turbulence due to averaged motion and the additional dissipation due to fluctuational slip of the phase may lead to both an increase and a decrease in the rate of pulsative motion of a carrier phase with particles (τ_u, $\tau_\theta \sim T_E$) [5, 6]. For fine particles with τ_u, $\tau_\theta \sim T_0$, the additional dissipation due to fluctuational slip of the phases approaches zero (f_{u4}, $f_{\theta4} \to 0$ at Ω_u, $\Omega_\theta \to 0$). However, in connection with the participation of particles in small-scale motion (g_{u1}, $g_{\theta1} \sim 1$), there is an increase in their contribution to turbulent dissipation and exchange interaction. It should be noted that for particles with τ_u, $\tau_\theta \ll T_\eta$, Eqs. (23) and (25) coincide with the equations for the intensity of velocity and temperature fluctuations of a one-phase turbulent flow in the case of high turbulent Reynolds numbers — when molecular transport can be ignored.

The second moments of the velocity and temperature fluctuations of the disperse phase in Eqs. (5) and (6) describe momentum and heat transfer resulting from entrainment of particles in the turbulent motion of the energy-containing moles. Expressions were found in [5, 6] for the second moments of velocity and temperature fluctuations of the particles in terms of the moments of the velocity and temperature fluctuations of the carrier phase.

3. To describe the dissipative and exchange terms in Eqs. (23)-(25), we use the approximate hypotheses of Rotta [12] and Monin–Kolovandin [13, 14]. Using the balance equations for the mean momentum and heat of a gas-suspension (5), (6), the equations for the energy of pulsative motion, and the square of the temperature fluctuations of the carrier phase (23), (25), we calculated the hydrodynamics and heat transfer for the steady turbulent motion of a disperse flow with a constant particle concentration in a circular pipe. The expressions for the eddy viscosity coefficient and diffusivity are found from Eqs. (23) and (24) in a nondiffusive approximation. The time macro- and microscales T_E and T_0 of turbulence are determined as in [8].

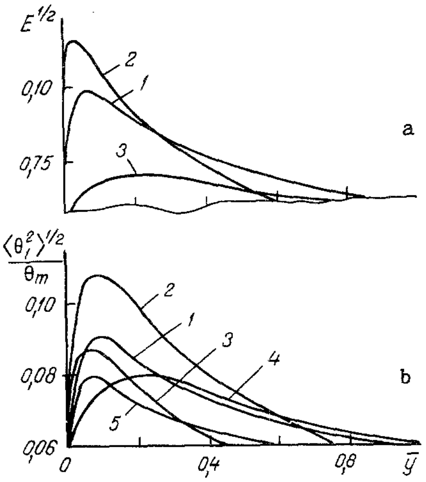


Fig. 1

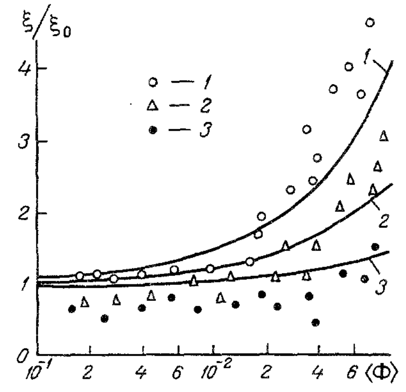


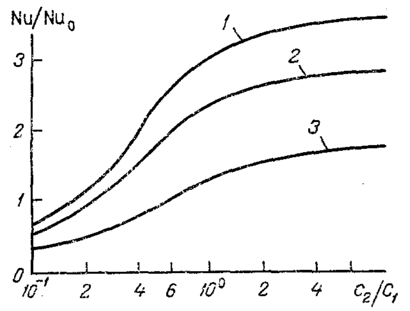
Fig. 2

Fig. 1. Effect of particles on the intensity of velocity fluctuations (a) and temperature fluctuations (b) ($Re = 5.3 \cdot 10^4$): a: 1) $\langle\Phi\rangle = 0$; 2) $\langle\Phi\rangle = 5$; R/a = 30,000; 3) 5; 5000; b: 1) $\langle\Phi\rangle = 0$; 2) $\langle\Phi\rangle = 5$; $c_2/c_1 = 0.5$; R/a = 30,000; 3) 5; 2; 30,000; 4) 5; 0.5; 50,000; 5) 5; 2; 5000.

Fig. 2. Comparison of experimental [15] and theoretical values of the drag of a gas suspension flowing in a circular pipe ($Re = 5.3 \cdot 10^4$): 1) R/a = 5000; 2) 3400; 3) 1700.

The completed calculations showed a reduction in the gradients of the mean velocity and temperature in the core of the disperse flow compared to a one-phase flow. This result is consistent with the experimental data in [15, 16]. As a consequence of the reduction in the gradients, there is also a reduction in the rate of generation of pulsations of the carrier phase in the flow core and an increase in the generation of pulsations near the wall of the pipe. Low-inertia particles ($\tau_u \lesssim T_0$) entrained in the small-scale pulsative motion increase turbulent dissipation, which in turn, reduces the rate of pulsative motion of the carrier phase in the core. In the neighborhood of the wall, there is an increase in the rate of pulsations of the carrier phase due to the additional generation of turbulent energy (curves 1 and 2 in Fig. 1a). An increase in particle inertia is accompanied by a reduction in the contribution of particles to turbulent dissipation. For particles with $\tau_u \sim T_E$, the reduction in the turbulent pulsations of the carrier phase is caused by fluctuational slip of the energy-containing moles of the carrier and disperse phases (curve 3 in Fig. 1). The reduction in the intensity of turbulent pulsations due to the work done by the flow in entraining particles with $\tau_u \sim T_E$ into pulsative motion is less than the reduction in the turbulence level due to the increase in turbulent dissipation for low-inertia particles (max $f_{u4} < 1$). This leads to a situation whereby the level of pulsations of the carrier phase in the flow core for particles with $\tau_u \sim T_E$ is higher than the level in the flow with particles having a relaxation time on the order of T_0. Figure 2 compares theoretical and experimental data [15] on the drag of a dust-laden flow in a circular pipe. The greater drag of the disperse flow compared to the one-phase flow is attributable to an increase in momentum transfer resulting from entrainment of particles in the pulsative motion of energy-containing moles.

The structure of the temperature fluctuations in a gas suspension depends both on the ratio of the dynamic and thermal relaxation times of the particles to the time scale of turbulence and on the ratio of the thermophysical properties of the particle material and carrier phase (Fig. 3). The greater the participation of the particles in the pulsative motion of the carrier phase and the greater the heat capacity of the particle materials, the smaller the gradient of mean velocity in the flow core. A reduction in the temperature gradient causes a reduction in the generation of turbulent pulsations of temperature in the core and an increase in pulsations near the channel walls. Low-inertia particles ($\tau_u \sim T_0$) lead at $c_2/c_1 > 1$ to a reduction in the intensity of gas temperature fluctuations in the core compared to a one-phase flow; at $c_2/c_1 < 1$, there is an increase in fluctuations of the temperature of the carrier phase (curves 1, 2, and 3 in Fig. 1b). The more inertial particles ($\tau_u \sim T_E$) cause a reduction in gas temperature fluctuations due to fluctuational phase temperature slip (Fig. 1b, curves 4 and 5). It should be noted that an increase in



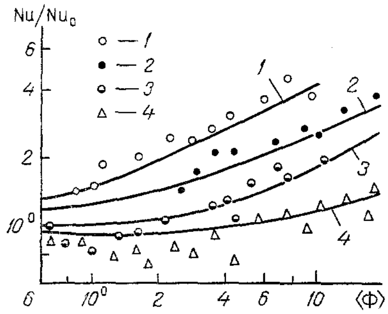


Fig. 3 Fig. 4

Fig. 3. Effect of the ratio of the heat capacities of the materials of the disperse and fluid phases on the Nusselt number of a disperse turbulent flow (Re = $3 \cdot 10^4$; Pr = 0.7; <Φ> = 5): 1) R/a = 5000; 2) 3000; 3) 2000.

Fig. 4. Comparison of experimental [18] and theoretical values of the Nusselt number of a gas suspension flowing in circular pipes: 1) Re = 10^4; R/a = 2500; 2) $1.2 \cdot 10^4$; 1100; 3) $1.35 \cdot 10^4$; 600; 4) $1.5 \cdot 10^4$; 200.

the dynamic relaxation parameter of the particles is accompanied by a reduction in the effect of the thermophysical properties of the disperse phase on the intensity of turbulent heat transfer.

Figure 4 compares the results of calculation of the Nusselt number of a gas suspension with experimental data [18]. As in the case of hydrodynamics, an increase in the rate of heat transfer by the disperse flow involves the entrainment of particles in pulsative motion.

Thus, we have proposed a method of calculating hydrodynamics and heat transfer for disperse turbulent flows in channels within a fairly broad range of dimensions, disperse-particle concentrations, and ratios of the thermophysical properties of the particle material and carrier gas.

NOTATION

$U_i(x, t)$, $V_i(x, t)$, actual velocities of the fluid and disperse phases; $\Theta_1(x, t)$, $\Theta_2(x, t)$, actual temperatures of the fluid and disperse phases; $V_{pi}(t)$, $\Theta_p(t)$, $R_{pi}(t)$, velocity, temperature, and coordinate of the p-th particle; ω, volume of the p-th particle; Ω_N, volume of the flow containing the N-th particle; ν, χ, molecular viscosity and diffusivity of the carrier gas; $\tau_u = (2\rho_2 a^2)/(9\rho_1 \nu)$, dynamic relaxation time of the particles; $\tau_\theta = (\rho_2 c_2 a^2)/(3\rho_1 c_1 \chi)$, thermal relaxation time of the particles; <C(x, t)>, mean volume concentration of the solid phase; δ(x), three-dimensional Dirac function; $E = (\langle u_x^2\rangle + \langle u_y^2\rangle + \langle u_z^1\rangle)/2$, fluctuation energy of the gas; $\langle n_1(x, t)\rangle$, numerical concentration of one particle; ρ_1, ρ_2, densities of the material of the fluid and solid phases; k, wave vector; ε, turbulent dissipation of the velocity fluctuations; $T_\eta = (\nu/\varepsilon)^{1/2}$, Kolmogorov time scale; $\ell_\eta = (\nu^3/\varepsilon)^{1/4}$, Kolmogorov space scale; $\Omega_u = \tau_u/T_E$, $\Omega_\theta = \tau_\theta/T_E$, parameters of the dynamic and thermal inertia of the particles; Pr = ν/χ; $Re_E = LE^{1/2}/\nu$, turbulent Reynolds numbers; L, spatial integral scale of turbulence; R, pipe radius; $Re = 2RU_m/\nu$, Reynolds number of the flow; U_m, mean mass flow velocity; Θ_m, mean mass flow temperature; $\bar{y} = y/R$; y, distance reckoned from the channel wall; ξ_0, ξ, drag of one- and two-phase flows; Nu_0, Nu, Nusselt numbers of one- and two-phase flows; $\langle\Phi\rangle = \rho_2/\rho_1\langle C\rangle$.

LITERATURE CITED

1. A. A. Shraiber, V. M. Milyutin, and V. P. Yatsenko, Fluid Mechanics of Two-Component Flows with a Disperse Solid [in Russian], Kiev (1980).
2. L. B. Gavin and V. A. Naumov, Inzh.-Fiz. Zh., 44, No. 6, 927-932 (1983).
3. F. N. Lisin and I. F. Guletskaya, Inzh.-Fiz. Zh., 48, No. 5, 733-737 (1985).
4. C. P. Chen and P. E. Wood, AIChE J., 32, No. 1, 163-166 (1986).
5. I. V. Derevich, V. M. Eroshenko, and L. I. Zaichik, Inzh.-Fiz. Zh., 45, No. 4, 554-560 (1983).
6. I. V. Derevich and L. I. Zaichik, Inzh.-Fiz. Zh., 45, No. 4, 554-560 (1983).

7. I. V. Derevich, V. M. Eroshenko, and L. I. Zaichik, Heat and Mass Transfer-VII [in Russian], Minsk (1984), pp. 141-146.
8. I. V. Derevich, V. M. Eroshenko, and L. I. Zaichik, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1, 40-48 (1985).
9. A. S. Monin and A. M. Yaglom, Statistical Fluid Mechanics, Part 1, Moscow (1965); Part II (1967).
10. I. V. Derevich, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1, 69-74 (1987).
11. J. O. Hinze, Turbulence, McGraw-Hill (1960).
12. J. Rotta, Z. Phys., 129, 547-572 (1951); 131, 51-77.
13. A. S. Monin, Fiz. Atmos. Okeana, 1, No. 1, 45-49 (1965).
14. B. A. Kolovandin and V. E. Aerov, Heat and Mass Transfer [in Russian], Vol. 2, Minsk (1969), pp. 66-87.
15. T. Tsuji, Y. Morikawa, and H. Shioni, J. Fluid Mech., 139, 417-434 (1984).
16. B. I. Katorgin, V. A. Perevezentsev, and V. A. Levchenko, Izv. Vyssh. Uchebn. Zaved., Energ., No. 12, 92-95 (1985).
17. P. G. Boothroyd, Trans. Inst. Chem. Eng., 44, No. 8, 306-313 (1966).
18. A. S. Sukomel, F. F. Tsvetkov, and R. V. Kerimov, Heat Transfer and Drag in the Motion of a Gas Suspension in Pipes [in Russian], Moscow (1977).

VORTEX FLOWS WITH SUSPENDED SEPARATION REGIONS AND LONG-RANGE UNTWISTED CENTRAL JETS

G. N. Abramovich and R. S. Trofimov

UDC 532.517.4

A study is made of possible physicoaerodynamic configurations of vortical flow with suspended separation regions and untwisted central jets. Such flows are encountered in power plants (combustion chambers, heat exchangers, chemical reactors, etc.) and in nature (tornadoes).

Special vortical flows with suspended separation regions and long-range untwisted jets occur in power plants — such as in certain types of heat exchangers, separators, high-speed combustion chambers, and chemical reactors — and in nature. The development of a theory of such flows needs to be backed up by qualitative experimental studies of the corresponding initial aerodynamic schemes.

This article is an attempt to describe the basic configurations of several flows of this type. The configurations may be useful in constructing methods of theoretical calculation.

1. Structure of a Flow Formed by Coaxial Cocurrent Twisted Jets. Coaxial cocurrent jets are employed in high-speed combustion chambers, chemical reactors, and mixers. Such flows have been studied in detail for the case of the absence of preliminary twisting. An example is the study in [1] with reference to ejectors.

Many investigations have focused on the case of a central twisted jet [2]. It was shown in [2] that a suspended separation region forms at the beginning of the central jet for sufficiently intense twisting. S. Yu. Krasheninnikov proposed a dimensionless criterion for the formation of this region. Quantitative studies have also been made of the case when both jets are twisted [3, 4]. However, no one has yet come up with a clear scheme for the formation of suspended separation regions in flows of this type. Nonetheless, suspended separation regions are necessary in high-speed combustion chambers to stabilize the flame and organize a stable diffusion front for the flame.

There are four cases in which suspended separation regions can be formed: 1) the jets are twisted in opposite directions; 2) the jets are twisted in one direction; 3) only one jet is twisted; 4) neither jet is twisted.

The translational velocities in jets 1 and 2 (Fig. 1) are usually different ($u_{1a} \neq u_{2a}$). The presence of twisting is connected with a reduction in pressure toward the axis of the

Sergo Ordzhonikidze Moscow Aviation Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 53, No. 5, pp. 751-757, November, 1987. Original article submitted March 5, 1987.